Mixing of dense fluid in a turbulent pipe flow

Part 1. Overall description of the flow

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This paper concerns an investigation into the behaviour of a layer of dense salt solution on the floor of a sloping rectangular pipe in which there is a turbulent flow. The various phenomena which are observed are described qualitatively and by the presentation of typical concentration profiles.

Numerical values are obtained experimentally for the rates of spread of the edge of the layer in the case where the salt is moving entirely in the direction of the main stream. The rate of spread is found to depend mainly on the slope α and on the pipe Richardson number, defined by $\mathrm{Ri}_p = D\Delta_d \cos\alpha / V^2$, where D is the depth of the pipe, $\Delta_d = g(\rho_d - \rho_a)/\rho_a$, ρ_d is the density of the fully mixed discharge and ρ_a is the density of the ambient flow. In the range of Ri_p from 0 to 0·005 the rate of spread of the layer decreases by a factor of about 3 at small slopes.

Some discussion is given of the factors determining the initial rate of spread just after the layer leaves the slit. Finally, it is shown how the depth measurements can be related to the determination of the concentration at the floor.

1. Introduction

In an earlier paper (Ellison & Turner 1959, hereafter called I) we have discussed the turbulent flow down a slope of a layer of fluid which is heavier than its surroundings. Such flows occur commonly in nature, examples being katabatic winds and turbidity currents in the oceans. We were able to characterize the mixing of these layers by the entrainment E, defined as the ratio of the velocity of inflow of the still or smoothly flowing ambient fluid into the turbulent layer to the relative velocity between the layer and its surroundings. Provided the Reynolds number was large, E was found to depend mainly on the overall Richardson number of the layer $\mathrm{Ri} = g(\rho - \rho_a) h \cos\alpha / \rho_a V^2$, where h is the depth of the layer and V its velocity relative to the ambient fluid. Once the function $E(\mathrm{Ri})$ had been determined empirically, it was possible to make certain theoretical predictions about, for example, how great a flow in the ambient fluid in the direction opposing gravity would be needed to cause the flow in the layer to be reversed.

Our previous calculations were, however, necessarily restricted to the case where the ambient fluid was deep and in laminar motion. There are many practical problems where the ambient fluid is contained in a pipe or channel and is turbulent for which analogous information is required. The situation is then much more complicated, since more variables are involved, and it is clear that the state of turbulence in both parts of the flow will be important. In this paper we report a series of experiments in a rectangular pipe which demonstrate qualitatively the various phenomena which can occur and provide quantitative information on the rate of mixing in such a channel as a function of a suitable stability parameter. The dependence on stability should have a similar form in other situations, but for reasons which will be discussed later, the numerical values which we find do not have the universal validity which is to be expected for the function $E(\mathrm{Ri})$ used in I. Great caution should be used in applying them to pipes of different shapes from ours or to cases where the ambient flow has a very different Reynolds number.

This investigation forms part of a more general study of the process of mixing in a turbulent fluid when there is a density gradient present. The information given here on the rate of spread of the layer is of direct practical interest and also indicates the values of the Richardson number at which the stabilizing effect of the density gradient becomes important, but in order to get to the heart of the problem it is necessary to consider the mechanism of the flow in more detail. In the second part of this paper simultaneous measurements of velocity and density will be present which can be used to interpret the mixing phenomena in terms of the dependence of the turbulent transfer coefficients on *local* stability parameters.

2. The apparatus

The discussion of the various observed phenomena will be simplified if we first describe the apparatus which was used. The experiments were carried out in a Perspex pipe of 10 by 5 cm rectangular section and $2\frac{1}{2}$ m long, mounted with

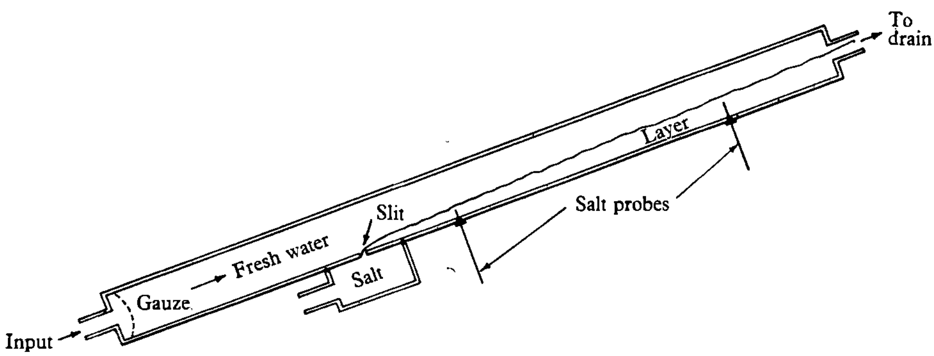

FIGURE 1. Sketch of the experimental pipe.

the 5 cm sides vertical (in order to have stratified flows as nearly two-dimensional as possible) on a tiltable Dexion frame. Water for the ambient or ventilating flow was supplied through a gauze screen at one end of the pipe from a constant head tank in the roof of the laboratory and flowed to waste at the other end. The drain pipe was raised to such a level that the static pressure in the experimental pipe was slightly positive as this facilitated the withdrawal of samples. Mean velocities of up to 15 cm s^{-1} could be maintained and were measured either by recording the time needed to fill a container of known volume or by timing the passage of patches of dye between fixed marks.

Salt solution was used to form the layer of denser fluid, and this was led from a roof tank to a Perspex box mounted against the lower face of the pipe. From this it flowed slowly through a slit 5 mm wide cut right across the face of the pipe about 1 m from one end (in the majority of the experiments the input end). The general arrangement is shown in figure 1. The rate of flow of salt solution was controlled by a series of nozzles, any one of which could be inserted in the supply pipe.

The techniques for determining the density profiles have been described in I. Provision was made at 10 cm intervals along the centre of the bottom face of the pipe for the insertion of fine stainless-steel tubes which could be traversed across the flow and through which samples of fluid could be obtained from any desired level; the density was measured by passing the samples continuously through a simple conductivity cell.

3. The types of flow observed

The various types of flow can be established depending on the slope of the pipe, the salt flux and the amount of ventilating water flow. We shall now describe these in turn, illustrating where appropriate with some quantitative measurements.

(i) *Ventilation with gravity (negative slopes)*

When the ventilating stream is in the same direction as the free flow of the layer with no ventilation, it is found that its turbulence has little effect on the qualitative behaviour of the system and the predictions of I for a laminar ventilating stream apply. As the ventilating velocity is increased, with slope and salt flux constant, mixing decreases until over an appreciable range of speed there is so little difference in velocity between the ventilating flow and the layer that the buoyancy forces become dominant and almost completely inhibit mixing. At very large ventilating velocities, when the layer is being dragged forward by the main stream, mixing again increases.

The dependence of mixing on slope is not very dramatic. One might expect it to decrease with increasing negative slope until at very steep slopes the $\cos\alpha$ term causes an appreciable reduction in the Richardson number (whose precise definition will be discussed in the next section) and an increase in the mixing. However, we found that at -20° the mixing was greater than at -10° and that breaking waves appeared on the top of the layer. It is unlikely that the layer is properly turbulent in either of these conditions (at the Reynolds numbers attainable in our pipe), and the effect is outside the scope of this paper though it deserves further study.

(ii) *Ventilation against gravity (positive slopes)*

When the water is flowing uphill, the picture is very different. Two cases may be distinguished according as the ventilating velocity is great enough to reverse the layer when it leaves the slit or not.

The simplest situation arises when the ventilating velocity is large, for then all the salt solution is carried uphill, forming a layer whose thickness increases with distance as it mixes across the channel. The rate of mixing depends quite strongly on the slope, since at steep slopes a greater turbulent shear stress is required to counteract the weight of the layer and drag it uphill: mixing is then increased both because the level of turbulence is increased and also because the local Richardson number is smaller and a given level of turbulence is likely to be associated with a larger diffusivity. This case of large ventilating flow when the layer is *fully reversed* is the one we have studied in most detail in §§4 and 5.

At rather lower rates of ventilation, part at least of the salt solution flows downhill against the water flow, gradually mixing as it does so. Thus in the part of the pipe downhill of the slit three regions of flow may be distinguished. First, there is a thin layer near the floor flowing downhill, whose depth decreases with distance from the slit as fluid is mixed out of it, but in which the concentration is

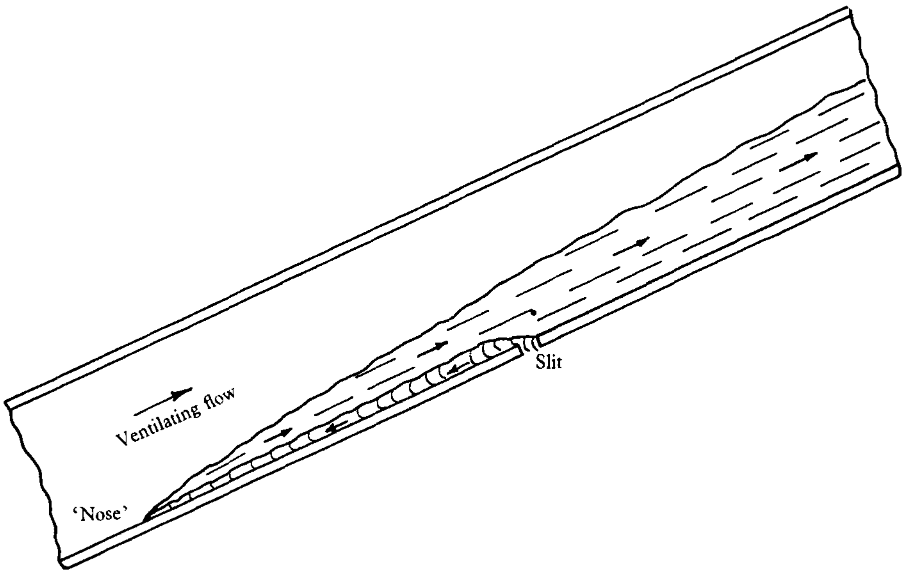

FIGURE 2. Sketch of the 'three-layer' type of flow which can be established at low ventilation velocities.

falling, showing that there is mixing in both directions across its boundary. This layer ultimately disappears, but in practice our channel was not always long enough to include the termination of the layer, and in these cases the part of the layer reaching the bottom end of the pipe was forcibly mixed near the fresh-water inlet. Above this downflowing layer is an intermediate region of mixed fluid flowing uphill and growing in thickness as it is swept up the channel; and above that, at the top of the pipe, is the pure ventilating stream. On the uphill side of the slit there is, of course, no layer flowing downwards and that part of the salt solution which moves uphill immediately it leaves the slit soon becomes part of the mixing intermediate region. The situation is shown diagrammatically in figure 2.

The great difference between the rates of mixing in a horizontal pipe and one tilted so that the three-layer flow is established is shown in figure 3. The two density profiles were obtained at the same station 20 cm uphill of the slit, and with identical flow rates for the fresh water and for the salt solution: the only difference lay in the slopes, which were respectively 0° and 13°.

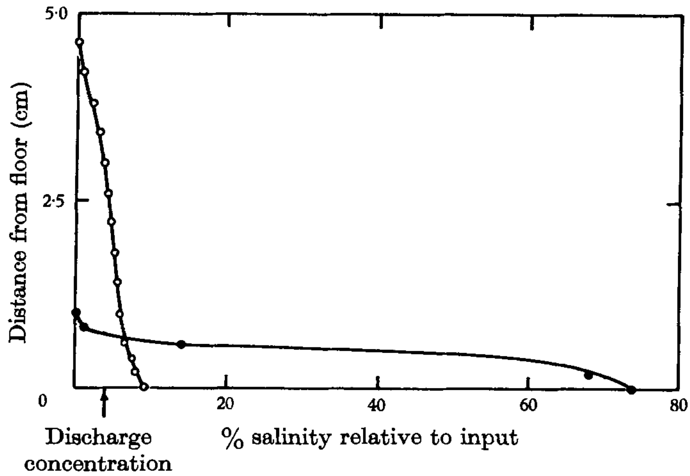

FIGURE 3. Density profiles at 20 cm uphill of the slit, showing the great differences between the mixing in a horizontal and an inclined pipe. $A/V^3 = 0{\cdot}017$. ○, Tilted at 13°; ●, horizontal.

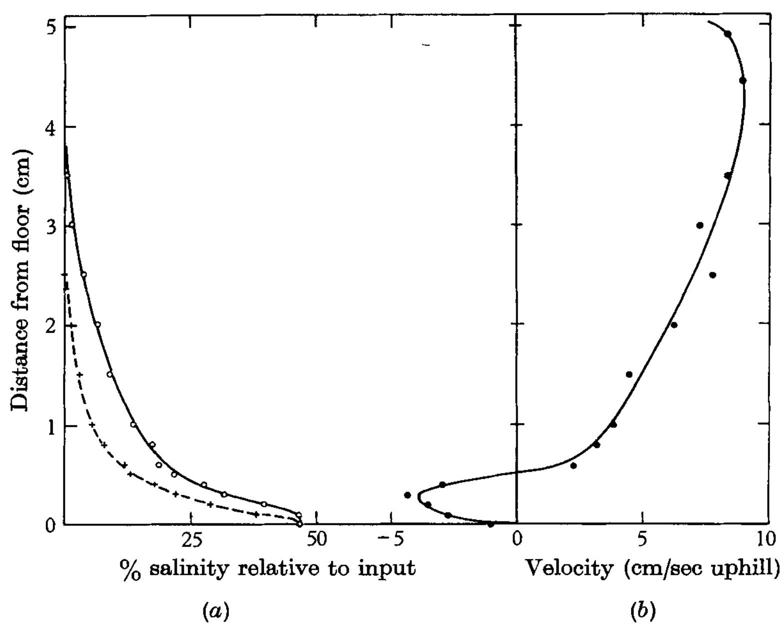

FIGURE 4. (*a*) Density profiles at 30 and 50 cm downhill in a 'three-layer' flow with the channel tilted at 23°. $\mathrm{Ri}_p = 0{\cdot}015$. (*b*) Velocity profile in the same flow, at 40 cm downhill. ○, salinity 30 cm from slit. +, Salinity 30 cm from slit. ●, Velocity 40 cm from slit.

The characteristic shapes of the density profiles downhill from the slit in the three-layer flow are shown in figure 4*a*. The two profiles are at 30 and 50 cm downhill from the slit with the channel tilted at 23°. The velocity profile at 40 cm is shown in figure 4*b*. This was obtained with thermistors and may not be very accurate, but it clearly shows the most important features. The greatest velocity downhill occurs near the top of the region of concentrated salt (where we would place the edge of the layer visually or from inspection of the density profiles alone). At a small distance above this is a region of very high velocity gradient, but even where there is only sufficient salt to change the density by a few parts in ten thousand the velocity profile is still distorted.

4. Fully reversed flows: basic ideas

When the whole of the salt solution is moving in the same direction there is hope that the dominant properties of the flow can be characterized simply by one or two quantities and a description of its development given in terms of overall parameters of the flow. Thus if h is a length specifying the depth of the layer (a precise definition will be discussed later) one would expect from dimensional reasoning that in a two-dimensional pipe of depth D, h/D would be a function of x/D, Re_p, Ri_p, α and Δ_0/Δ_d, where x is the distance from the slit, Re_p is the Reynolds number defined by $\mathrm{Re}_p = VD/\nu$, Ri_p is the 'pipe Richardson number' defined by $\mathrm{Ri}_p = D\Delta_d \cos\alpha/V^2 = A\cos\alpha/V^3$, α is the slope, and Δ_d and Δ_0 are related to ρ_d the discharge density (i.e. the density when the salt solution is fully mixed with the ventilating flow) and the density ρ_0 with which it is emitted by $\Delta_d = (\rho_d - \rho_a)g/\rho_a$ and $\Delta_0 = (\rho_0 - \rho_a)g/\rho_a$, V is the discharge velocity, A the rate of output of excess weight, which is constant in a given experiment, and ρ_a the density of the ambient flow. In attempting to apply this relation in practice, the dependence on Δ_0/Δ_d presents a special difficulty. There is inevitably a region of rapid mixing close to the slit, where the salt solution is being accelerated and one might expect that even if the rate of output of salt solution was always kept constant the effective Δ_0/Δ_d might depend critically on the details of the design of the slit. If this were so and h/D depended appreciably on Δ_0/Δ_d, measurements of the depth as a function of the other variables might have rather little value.

In order to avoid this difficulty, one might suggest that except very near the slit the flow would be in local equilibrium at each section with properties independent of its history. In that case one would have

$$\frac{\partial h}{\partial x} = f\left(\frac{h}{D}, \mathrm{Re}_p, \mathrm{Ri}_p, \alpha\right). \tag{1}$$

We did attempt to interpet our measurements, for which Re_p was constant = 6500, in terms of this formula, but we found inconsistencies suggesting that history did matter. If, however, instead of attempting to plot $\partial h/\partial x$ as a function of Ri_p and α, choosing the position x at which h/D has a particular value, we plot the difference between the values of h at two fixed positions, taken in our case to be at $x/D = 4$ and $x/D = 24$, we find much greater consistency. It is possible in practice to ignore variations in h/D except where this attains values comparable with $\frac{1}{2}$, and to obtain the rate of increase of depth as a function of Ri_p or A/V^3 and α alone. We shall therefore present our measurements in this way, discussing separately the manner in which h/D at $x/D = 4$ is determined by the initial conditions.

The definition of h

In I we were able to define the depth of a sloping plume from integrals of the velocity profile, but such a definition is clearly impossible here. So far as the dimensional argument is concerned, any dimensionally correct definition may be used and the choice is entirely one of convenience in measurement and practical application; if a detailed theory were available it might well influence the choice, but at present there is none. There are several possibilities. For example, one might define h as the height where the concentration is $1/10$ or $1/100$ the value it has at the floor at the same station. This definition would have several disadvantages: it would require two measurements to determine h since the way the floor concentration varies with x is not known *a priori* (indeed, the floor concentration is rather an unsatisfactory quantity, since it depends critically on the behaviour of any thin viscous sublayer which may be otherwise negligible) and it also breaks down at whatever value of x the flow becomes so mixed that the variation of concentration across the channel is less than the selected standard ratio.

We decided therefore to define h as the height where the concentration equals the discharge concentration Δ_d. This height can readily be determined since Δ_d is independent of x; it certainly always exists, and as will be shown later it is readily related empirically to quantities of direct practical importance. It does, however, suffer from its own disadvantages. The most serious of these is that the measurement of $\partial h/\partial x$ is sensitive to the accuracy of the determination of Δ_d, and in our channel, owing to its rectangular shape, the actual discharge value of Δ_d, which is what we used in the experiments, is not exactly the same as that for an ideal two-dimensional channel having the same flow as exists on the centre line of our pipe. One must also note that although h always exists, its value has little significance after it first reaches a value of $\frac{1}{2}D$. In most cases the layer is then for practical purposes mixed, but if it is of interest to study the final stages a different specification will be required.

In the next section our measurements will be presented in detail, indicating how the salt profiles and h change between $x/D = 4$ and 24. In the following section some discussion of the factors determining h at $x/D = 4$ will be given and then finally a few comments will be made on the relation between h and the other quantities of interest.

It must be emphasized that whatever criticisms may be made of our particular definition of h, the measurements of $\partial h/\partial x$ as a function of Ri_p and α do clearly indicate the values of these quantities which produce significant changes in the rate of mixing. These were not previously known even approximately: and they should be little changed by changes in the scale and shape of the channel, which will probably alter the numerical values for $\partial h/\partial x$.

5. Measurements on fully reversed flows against gravity and on flows in the same direction as gravity

All the measurements reported in this section have been obtained using the same rate of ventilating flow; this was the maximum which could be conveniently maintained with the available water supply and corresponded to a pipe Reynolds number based on depth and discharge velocity of about 6500. We have not studied the variation of mixing with Re_p as we thought it best to concentrate on obtaining a broad picture of the phenomenon by operating the highest possible Reynolds number. However, except in certain cases at high values of Ri_p when the

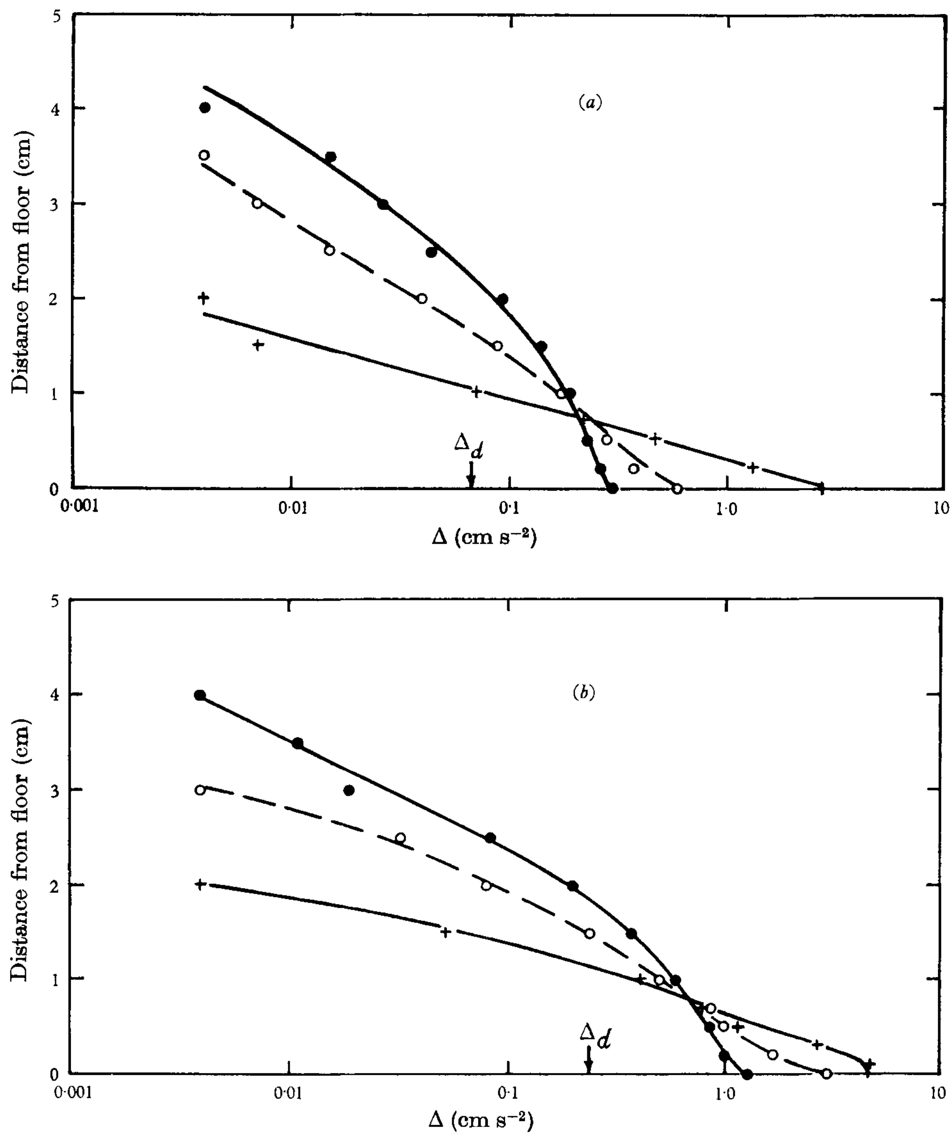

FIGURE 5. Density profiles in fully reversed flow, slope $+10°$. +, 20 cm uphill; ○, 70 cm uphill; ●, 120 cm uphill. (*a*) $Ri_p = 0{\cdot}0018$. (*b*) $Ri_p = 0{\cdot}0068$.

mixing is very small and complicated problems of stability arise, it seems plausible to suggest that the mixing in a given distance will be proportional to the turbulent intensity which is roughly proportional to $C^{\frac{1}{2}}_D$; hence since C_D, the drag coefficient of the pipe, is a slow function of Re, the mixing will only depend on Re weakly.

The differences between laboratory measurements in smooth pipes and large scale phenomena in rough pipes (e.g. mine galleries) should also be small since C_D is not very different in the two cases.

(i) *Profile measurements*

For comparison with the measurements recorded in §3 we first present some density profiles which illustrate in outline the effects we have been discussing. In figures 5*a* and *b* are shown density profiles taken at 20, 70 and 120 cm uphill from the slit with the pipe sloping at 10°. A logarithmic scale is used for Δ in

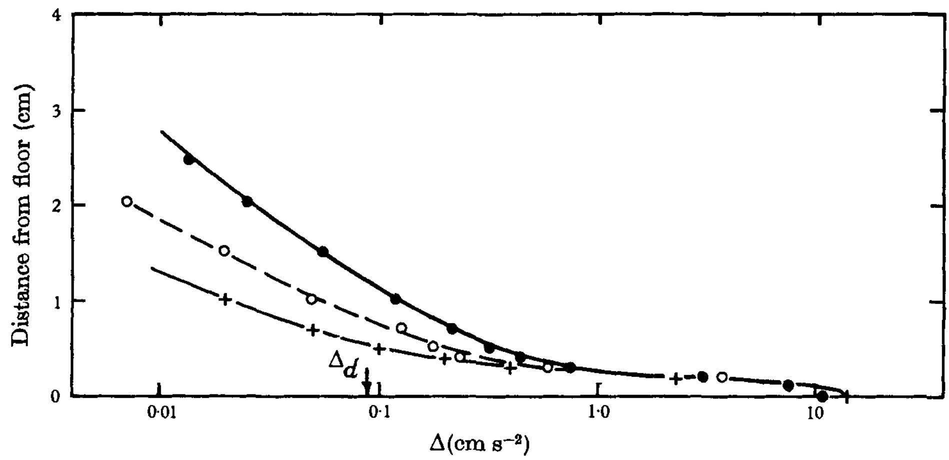

FIGURE 6. Density profiles in fully reversed flow, slope + 5°, $\mathrm{Ri}_p = 0{\cdot}0025$. +, 20 cm uphill; ○, 70 cm uphill; ●, 120 cm uphill.

order to cover the wide range observed, and the measured discharge values are shown; they correspond to $\mathrm{Ri}_p = 0{\cdot}0018$ in figure 5*a* and 0·0068 in figure 5*b*. It is apparent that the layer with the higher Richardson number spreads less rapidly than the other; the absolute depths are not very different in the two cases.

In figure 6 are shown profiles at the same stations as before for an experiment with $\mathrm{Ri}_p = 0{\cdot}0025$, much the same as that in figure 5*a*, but with the slope 5°. The rate of spread is much less at the lower slope. Note that there is also a very thin region near the wall containing rather concentrated salt which is almost certainly governed by viscosity. Viscous layers such as this are unlikely to carry an appreciable fraction of the salt flux except where the mixing is extremely small, so the behaviour of the outer edge of the layer should be little changed, but we should remember that the dependence on Reynolds number is likely to be more serious at low slopes.

The measurement of full profiles is tedious, so frequently we have determined h alone by first measuring the discharge concentration and then finding by trial and error two points at each cross-section having mean concentrations straddling this value and between which we could interpolate. This technique has enabled us to conduct an extensive series of experiments at different slopes and Richardson numbers, and the results are shown in the next section.

(ii) *The dependence of the growth of the layer on stability and slope*

All our measurements of the difference between h/D at $x/D = 4$ and $x/D = 24$ with the pipe at the fixed slope of $+10°$ are plotted in figure 7 as a function of A/V^3. The mean absolute values of h/D are shown. It should be noted that although we chose not to measure the rates of spread at a fixed height, our results cover a fairly narrow range of mean layer depths with h/D between 0·2 and 0·4; within this range there is no systematic variation of the rate of spread with h/D.

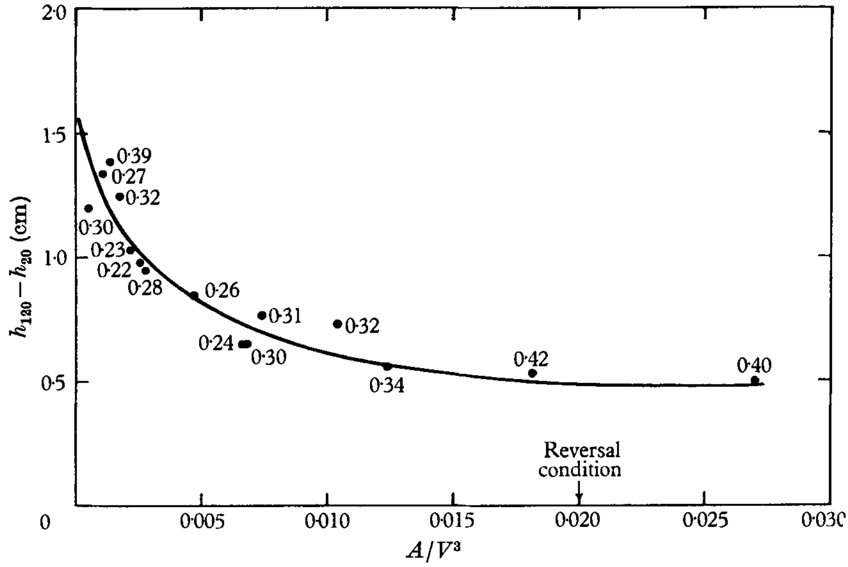

FIGURE 7. Experimental values for the increase in depth of the layer between $x/D = 4$ and $x/D = 24$ at a fixed slope of $+10°$, as a function of A/V^3. The mean values of h/D are marked against the points.

Also marked on figure 7 is the reversal condition taken from our measurements which were previously published in I; points to the right of this correspond to layers which had some nose extending downhill.

We did also attempt to measure the spread of unstable layers by inverting the channel so that the slit was in the top, but as might be expected the layer disintegrated in all cases well before $x/D = 24$ was reached. All that can be said is that over the range $x/D = 4$ to $x/D = 8$ the rate of spread was much greater than it is in the stable case.

Graphs similar to figure 7 have been prepared for other slopes, but no useful purpose would be served by presenting all the experimental points in detail here. Instead, smooth curves have been drawn through each set of points; these are combined in figures 8*a* and *b*, and constitute the main results of this paper. The general trend of these curves and the variation with slope is much as expected from the previous discussion. Note, however, that as we mentioned briefly in § 3, the mixing at the downhill slopes of $-20°$ and $-30°$ is greater than it is at $-10°$. Such an increase would be expected at very steep slopes where the layers would be pulled forward by gravity at a velocity greater than the stream above, but in this case a different mechanism is thought to operate. Breaking waves, rather like roll waves, were observed on the interface and these probably were responsible for transferring the salt solution upwards.

At very low Richardson numbers all the curves come together (within the experimental error), as, of course, they must when density differences are negligible. The value of the rate of spread in neutral conditions has to be found by

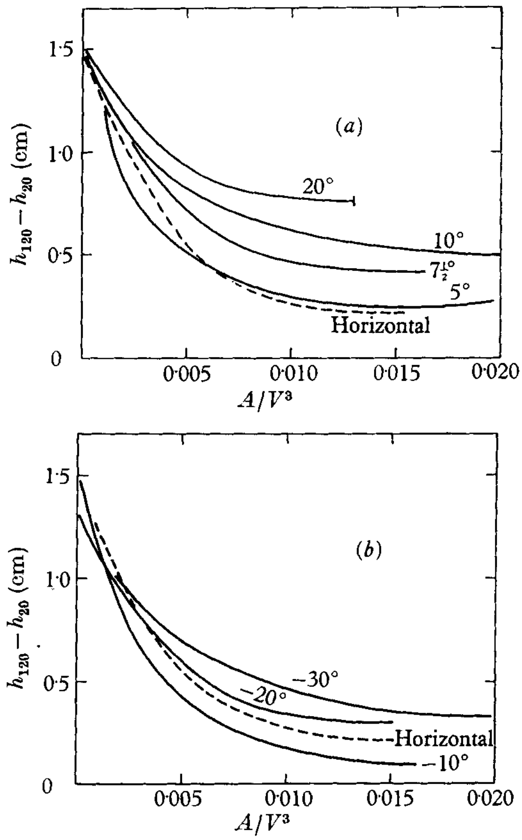

FIGURE 8. Mean experimental curves showing the spread of the layer as a function of A/V^3 at various slopes. (*a*) Fully reversed layers. (*b*) Layers in the same direction as gravity.

extrapolation, and is about $1\cdot5 \times 10^{-2}$. We were unable to make measurements at Richardson numbers lower than about 0·0005 because the discharge concentration was then only about twice that in the fresh-water supply. In an attempt to obtain more accurate results in neutral conditions we made a conducting solution of neutral buoyancy by mixing salt solution with methylated spirits, but we were unable to use such a mixture, since its density does not remain equal to that of water when it is diluted by further mixture with the ventilating flow the rates of mixing in normal and inverted experiments were quite different.

In neutral conditions the diffusing solution ought not to affect the flow in the pipe and if one made the usual assumption that the *Austausch* coefficients for momentum and salinity are equal (Reynolds analogy) or in a known ratio, there is no reason why a theoretical calculation of the concentration profiles in neutral conditions could not be carried out, since K_M is sufficiently known. We have not been able to find such a calculation in the literature, although Calder (1949) has given a theory of diffusion from a line source in a constant stress boundary layer in which he approximates to the logarithmic velocity profile by a power law. This theory can only be applied to flow in a pipe when none of the pollutant has reached half way across and this restriction prevents us from using it in our case except at small values of x/D. Here Calder's theory gives profiles in broad agreement with our measurements, but seems to predict a rate of spread greater than we observed.

6. The initial mixing

As we have mentioned earlier, the rapid mixing near the source is very important in determining the depth of the layer at $x/D = 4$; and indeed this depth is often much greater than the increase in depth between $x/D = 4$ and $x/D = 24$. Detailed attempts to explain quantitatively our results for this region have not been successful, but a discussion of some factors which must be involved in any full theory may perhaps stimulate someone to clarify the position. This leads us away from our main theme, but since we are in this paper aiming to present results in a form in which they can usefully be applied, it seems undesirable to ignore the initial mixing and so to imply that measurements of the steady state mixing solve the whole problem.

The initial mixing takes place while the layer is accelerating to its quasi-equilibrium velocity, and in order to determine its depth at the end of this process we must consider not only the acceleration itself, but also the way in which the depth and velocity of the unmixed layer are determined when it leaves the slit.

While the layer is thin and the velocity difference between it and the ambient stream is large, the turbulence in the ambient stream is unlikely to have much influence and the methods of I may be used subject to some reservation concerning the ability of the turbulence to adjust itself quickly enough to maintain a state of changing equilibrium. A calculation based on this idea shows that the layer accelerates rapidly, at first becoming thinner and then after a very short distance beginning to thicken again. When the channel is sloping a calculation of the final depth requires a detailed knowledge of the dependence of entrainment on 'layer Richardson number'; but, for a horizontal channel, if we neglect friction and any change in the velocity of the ambient stream which may be produced if the layer is not thin enough, it is easily seen that one relation between the velocity and depth of the layer before and after mixing may be obtained from consideration of momentum alone. In fact for rectangular (top-hat) velocity profiles

$$V_0(V_a - V_0)\,h_0 = V_1(V_a - V_1)\,h_1, \tag{2}$$

where V_0 and V_1 are respectively the velocities of the layer before and after the acceleration and V_a is that of the ambient stream. A second relation is to be found from the condition that the acceleration will have ceased when the Richardson number approaches its critical value (which we may take from I to be about 0·8).

There remains the difficulty that in order to find the final depth, the initial depth and velocity have to be specified. The manner in which they are determined is very complicated and we can envisage at least three different mechanisms which can operate.

First, one may consider a layer flowing downhill with only a low ventilating velocity. In this case the velocity and depth very near the slit are determined by the condition that long gravity waves on the interface should be stationary, just as in the case of the flow of water over a weir. The expression for the wave velocity on the interface between two moving streams of finite depth is well known (Lamb 1932, §234) and it shows that stationary waves can only exist provided that

$$\Delta_0 > V_a^2/D\cos\alpha, \tag{3}$$

and that if h_0 is small compared with D, it is given by

$$\left(\frac{h_0}{D}\right)^3 = \frac{(\Delta_d/\Delta_0)^2}{[(\Delta_0/\Delta_d)\,\mathrm{Ri}_p - 1]}. \tag{4}$$

Once h_0 is known, the value of V_0 can be found from the given values of A and Δ_0.

At high values of the ventilating velocity or low values of the initial density, the condition (3) is not satisfied and a second mechanism must operate. In this the saline water issuing from the slit is initially bent forwards by the pressure of the oncoming stream on the interface. One may make a very crude theory of this by assuming that the initial depth is determined in a distance so short that the component of the weight of this part of the layer along the slope, mixing across the interface and bottom friction can all be neglected. If this is so, one can apply the law of conservation of momentum to the whole flow in the pipe before and after the slit and also Bernouilli's equation to the flow of fresh water. These equations lead to

$$\left(\frac{h_0}{D}\right)^3 = \frac{2(\Delta_d/\Delta_0)^2}{[1 - (\Delta_0/\Delta_d)\,\mathrm{Ri}_p]}. \tag{5}$$

Unfortunately not even (4) and (5) are sufficient to cover all situations; when the flow is uphill the weight of the layer may have serious consequences which invalidate the two theories. The turbulence on the interface takes a short distance to develop and the weight of the layer over this distance is unbalanced and causes the layer to flow back over the slit forming an embryonic version of the three-layer flow described in §3(ii), with a nose a centimetre or so downhill. In this situation (which is the rule rather than the exception) there are obviously many complications—for example, the nose may be so thin that viscosity is important—and we have not found any simple description of it.

However, this last difficulty does not arise when the channel is horizontal, and we have calculated the values of h_0 for that case using (4) and (5) and taking $\Delta_d/\Delta_0 = 5\times10^{-3}$, and then h_1 using (2) and the assumption that the acceleration is complete when the layer Richardson number is 0·8. The results are plotted in figure 9 and may be compared with a curve representing the average of a series of measurements in our channel. The theoretical values are very sensitive to the assumptions which have been made, but the theory at least gives the right form of variation. At high uphill slopes the measured h_1/D are found to increase with increasing Ri_p, an effect which must be associated with the complications discussed in the last paragraph.

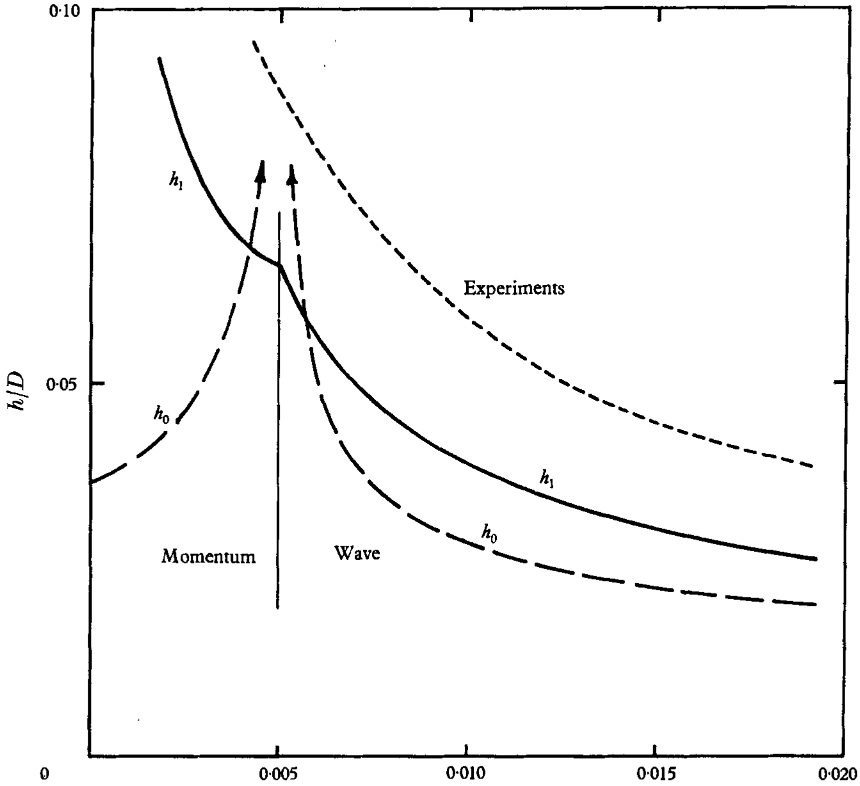

FIGURE 9. Comparison of theoretical estimates of the thickness of the layer at the slit and immediately after the initial mixing in a horizontal channel with measurements near the slit.

7. The relation between *h* and the floor concentration in dilute layers

The maximum concentration in the layer, which occurs at the floor, is often of practical interest. We decided against the floor concentration as an adequate parameter to describe the whole development of the layer, because of the possibility of a thin concentrated viscous region at the wall, but this sublayer disappears as the salt spreads out. After the layer has become diluted the concentration profiles do not depend much on other properties of the flow and it is thus possible to determine experimentally a unique relation between the maximum concentration Δ_s expressed as a multiple of the discharge concentration, and h. This is presented in figure 10, where we have taken mean values from a dozen experiments for which concentration profiles were available and which covered a wide range of Ri_p and slopes of from 5° to 40°. The experimental relation can be represented with good accuracy over the range of the measurements by

$$\log_{10}(\Delta_s/\Delta_d) = 3{\cdot}60 - 7{\cdot}1\,h/D. \tag{6}$$

This result enables us to predict the rate of decrease of the floor concentration in dilute layers as a function of A/V^3 and slope. Further, if we are given the depth of a layer somewhere before it has become dilute we can estimate the distance it must travel before the floor concentration falls below some specified multiple of the discharge concentration.

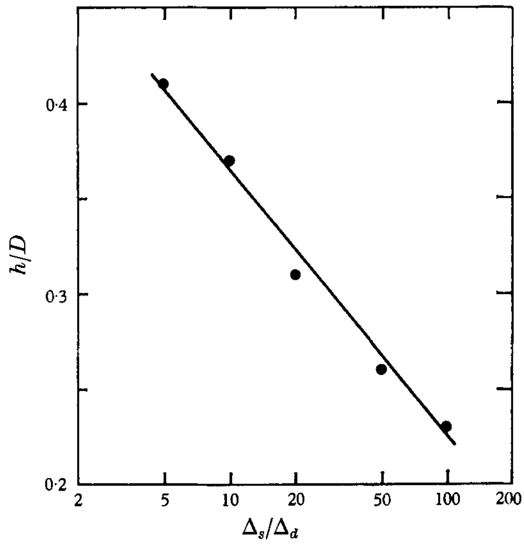

FIGURE 10. The experimental relation between the depth of dilute layers and the floor concentration.

To make the example specific, let us assume that $A/V^3 = 0{\cdot}01$ and that the slope is 10°. Suppose that at a known position $h/D = 0{\cdot}1$ and that we require to find how far from this the maximum concentration will be only 10 times the discharge concentration. From figure 7 the steady state dh/dx is seen to be $6{\cdot}0 \times 10^{-3}$, and from equation (6) we find that the final h/D is 0·365. Hence the required length is about $44D$.

The curves of figure 8 could have been drawn with this application in mind, and in special situations, where the conditions of emission of the layer are well defined, it would also be possible to include the effect of initial mixing and present results for the distance from the point of emission to points where the maximum concentration had certain standard values, as functions of Ri_p. However, because of the difficulties referred to in the last section we prefer to leave the results in their present form, which contains all the information which ought to be applied directly.

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